Paul Lintilhac

Problem Set 4 – Artificial Intelligence

You have a box with 6 coins.

3 of the coins are weighted so that they come up heads with probability 0.1. (Category 1)

1 is weighted so that it comes up heads with probability 0.6. (Category 2)

2 are weighted so that they come up heads with probability 0.7. (Category 3)

A. What is the probability of heads, if you pick a coin at random and ﬂip it?

B. Suppose that you pick three coins at random from the box and you ﬂip each of them 10 times.

What is the expected total number of heads? Justify your answer. (Hint: Use random variables.

This is easy , once you have done part (A). If you start enumerating the possible combinations of

categories, you are on the wrong track.)

Given that, we can consider expressing the total number of heads over all 30 coins as the sum of indicator function , which takes a value of 0 when the result of the i-th of the j-th coin |flip is tails, and 1 otherwise. We can express the expectation in terms of two nested sums: one sum that goes over the 3 chosen coins, and the other that goes over the 10 flips of each coin:

C. Suppose that you pick a coin at random and ﬂip it and it comes up heads. What are the

probabilities of each of the categories? What is the probability that it will come up heads again if

you ﬂip it again?

Category 1: By Baye’s Theorem,

The denominator above can be expressed using marginal expectations:

Using the answer to the previous problem, we can easily see that this probability is .

Thus,

Similarly,

and

Notice that becase of the denominator used in Baye’s theorem, these conditional probabilities are properly normalized so that they add to 1: .131 + .261+.609

Now, in order to find the probability that the next flip also turns up heads, we can simply calculate

D. Suppose that you pick a coin at random, ﬂip it twice, and it comes up heads both times. What

are the probabilities of each of the categories? What is the probability that it will come up heads

again if you ﬂip it again?

Similar to the last problem, we need to derive

Thus,

Similarly,

And

Again, notice that the probabilities sum to 1.

Now, in order to find the probability that the next flip also turns up heads, we simply calculate

E. Someone makes you the following oﬀer: You may pick a coin at random out of the box. You will

be allowed to place a $10 bet on the outcome of a ﬂip. (That is, they will pay you $10 if you win

and you will pay them $10 if you lose.) How should you bet? What is the expected payoﬀ of the

game?

$10

.1

Place bet?

**$0**

**-$8**

-$10

$10

.9

3/6

**-$2.5**

yes

-$10

**$2**

.6

1/6

no

**$4**

$10

.7

.4

2/6

$0

-$10

.3

There is only one choice involved with the game: play or don’t play. If you play, there are six possible scenarios: you can pick any of the 3 coins, and each of those coins can result in either a heads or a tails. I represent this as two separate levels of the tree above. In order to work our way backwards and calculate the expected value of the game, first we fill in the expected value for each coin. This is simply 10\*(P[heads])-10\*(P[tails]). Therefore, we can fill in the expected value at each of the blue triangles above. In order to calculate the expected value of the whole game, we simply calculate the average of the blue triangles, weighted by their respective probabilities:

Since the expected value of placing a bet is negative, you should not bet at all. Taking into account this choice to not place a bet, the expected value of the whole game is $0.

F. Now you get a better oﬀer. As in (E) you may pick a coin at random out of the box and you

will be allowed to place a $10 bet on the outcome of a ﬂip. However, before placing the bet, you

are allowed to ﬂip it once to test it. What is the proper strategy for placing the bet after you have

done the test ﬂip? What, at the start, is the expected payoﬀ from the game?

game?

In order to answer this question, we will need to use some of the findings from previous questions. First, recall that if we pick a coin out of the box at random, the probability of getting a heads is .383. Second, recall from problem C that once the coin has landed heads, the probability of getting another heads with the same coin is .596. Using the same kind of argument as in problem C, we can also find the probability of getting another heads given that the first flip was tails. Again using Baye’s theorem,

Note that

For category 1, this is

Now we can calculate

Place Bet? 

**$1.92**

**$1.92**

$10

H

.596

yes

T

.404

H

.383

Test Flip?

**$0.74**

**$0.74**

-$10

$0

No

yes

**-$4.96**

yes

H

.252

Place Bet?

**$0**

T

.617

No

T

.748

$10

$0 (game 1)

-$10

No

$0

Therefore, the proper strategy is to place the bet if the test flip is heads, and to not place any bet if the test flip is tails. The total value of this new game is $.74.

Problem 2:

A and E will be able to communicate as long as C is active, or if both B and F are active.

a)

b)

Using Baye’s theorem,

We already know the denominator from the answer to part 1, and we know . To find

, consider the fact that if F has failed, the only possible way for A and E to communicate is if C is active, which occurrs with probability .9. Therefore,

Problem 3:

According to the class notes, the reviewer in this problem has a probability of .7 of accepting a paper that would ultimately be accepted by the publication, and a .4 probability of rejecting the paper if it would ultimately be rejected by the publication.

Before calculating the decision tree, I will first calculate some probabilities that will be useful in the following problems.

First, we can use Baye’s Theorem to calculate the probability that the paper will be accepted if the reviewer accepts it:

We can calculate by expanding it using the law of marginal probabilities as follows:

Therefore

Note that we do not have to derive the conditional probabilities of success in the case where the reviewer rejects the paper, as mentioned in the lecture notes, because it would not make sense to seek the reviewers advice and then not follow it.

!. Consulting with one reviewer (dollars are in units of $1000 to save space):

$50

Submit? 

**$8.24**

**$8.24**

T

.304

yes

Reviewer accepts?

F

.696

T

.46

-$10

$0

Consult?

**$3.3**

**$3.8**

No

yes

F

.54

No

$2 (no reviewer)

$0

Therefore, the optimal strategy is to consult with the reviewer, and the expected value is $3300

2.

Now, to illustrate the case of when there are 2 identical reviewers, let denote the Boolean variable representing the first reviewers decision, and denote the Boolean variable representing the second reviewer’s decision. We want to consider the strategy where the author takes the reviewer’s advice only if both reviewers accept the paper.

The probability of being accepted given that both of the reviewers accept is:

Note that since the reviewers are conditionally independent of each other given the actual decision, we can write the joint conditional probability as

Note that and are not unconditionally independent. Rather in order to find the oint probability , we write

Therefore,

It makes sense that , because we are putting a stronger condition that both reviewers must accept, so we would expect a higher probability of the publication accepting given those conditions.

Submit? 

**$16.04**

**$16.04**

Both Reviewers accept?

$50

T

.434

yes

F

.566

T

.226

-$10

$0

Consult?

**$3.12**

**$3.62**

No

yes

F

.774

No

$2 (no reviewer)

$0

Thus the best strategy is still to consult, with the expected payoff using this strategy of $3120.

3.

Now, we look at the strategy where the authorpublished the paper only if one of the two reviewers accepts it. The probability of being accepted by the publication if either of the reviewers accepts the paper:

In order to find we can use the identity

And similarly we can find by noting that

Now we are prepared to calculate

It makes sense that this probability is less than the single reviewer case, because having one reviewer accept the paper is a stronger constraint than having one of two reviewers accept the paper, so it should say less about our chances of getting published.

Submit? 

**$5.72**

**$5.72**

Both Reviewers accept?

$50

T

.262

yes

F

.738

T

.694

-$10

$0

Consult?

**$3.47**

**$3.97**

No

yes

F

.306

No

$2 (no reviewer)

$0

Thus the best choice in this case is still to consult, with the expected payoff of this strategy being $3470.

Interestingly, the best strategy appears to be the last one (submit only if one of the reviewers accepts). It is interesting because the consultation says less about whether the paper will actually get published than the “AND” case – but evidently, this is counterbalanced by a higher probability of getting a positive review in the first place.